Differential Equations
Separation of Variables
Solutions of Practice Problems

Solution 1: Given that: $\frac{dy}{dx} = \sqrt{16 - x^2}$

Simplifying: $dy = \sqrt{16 - x^2}\, dx$

Integrating both sides, we have:

$$\int dy = \int \sqrt{16 - x^2}\, dx$$

$$\int dy = \int \sqrt{4^2 - x^2}\, dx$$

Using the formula:

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} sin^{-1}\frac{x}{a} + C$$

$$y = \frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2} sin^{-1}\frac{x}{4} + C$$

$$y = \frac{x}{2}\sqrt{16 - x^2} + 8\, sin^{-1}\frac{x}{4} + C$$

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Solution 2: Given that: $\frac{dy}{dx} = y\,sec^2 x$

Simplifying: $\frac{dy}{y} = sec^2 x\,dx$

Integrating both sides, we have:

$$\int \frac{dy}{y} = \int sec^2 x\,dx$$

$$\ln y = \tan x + k$$

Solution 3: Given that: $y^3 \frac{dy}{dx} = x^3$

Simplifying: $y^3 dy = x^3\,dx$

Integrating both sides, we have:

$$\int y^3 dy = \int x^3\,dx$$

$$\frac{y^4}{4} = \frac{x^4}{4} - c$$

$$y^4 - x^4 = 4c$$

$$y^4 - x^4 = k$$

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Solution 4: Given that: $\frac{dy}{dx} = \frac{1+y}{1-y}$

Simplifying: $\frac{1-y}{1+y}dy = dx$

Integrating both sides, we have:

$$\int \frac{1-y}{1+y}dy = \int dx$$

$$\int \frac{1+1-1-y}{1+y}dy = \int dx$$

$$\int \frac{2-(1+y)}{1+y}dy = \int dx$$

$$\int \frac{2}{1+y}dy - \int \frac{(1+y)}{1+y}dy = \int dx$$

$$2\int \frac{1}{1+y}dy - \int dy = \int dx$$

$$2\ln|1+y| - y = x + k$$

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Solution 5: Given that: $\frac{dz}{dt} = 3t^2 - te^{t^2}$

Simplifying: $dz = \left(3t^2 - te^{t^2}\right)dx$

Integrating both sides, we have:

$$\int dz = \int \left(3t^2 - te^{t^2}\right)dx$$

$$z = \int 3t^2 dx - \int te^{t^2} dx$$

$$z = 3\int t^2 dx - \int te^{t^2} dx$$

$$z = t^3 - \frac{1}{2}e^{t^2} + k$$